

AP CALCULUS AB SUMMER MATH 2020 - KEY

1] **COMPLEX FRACTIONS:** Simplify each of the following.

a] $\frac{\frac{25-a}{a}}{5+a}$ $\frac{5-a}{a}$ b] $\frac{4-\frac{12}{2x-3}}{5+\frac{15}{2x-3}}$ $\frac{4(x-3)}{5x}$ c] $\frac{\frac{x}{x+1}-\frac{1}{x}}{\frac{x}{x+1}+\frac{1}{x}}$ $\frac{x^2-x-1}{x^2+x+1}$

2] **SIMPLIFYING EXPRESSIONS:** Write answers with positive exponents only.

a] $\frac{\frac{2}{x^2}}{\frac{10}{x^3}}$ $\frac{x}{5}$ b] $\frac{12x^{-3}y^2}{18xy^{-1}}$ $\frac{2y^3}{3x^4}$ c] $(4a^{5/3})^{3/2}$ $8a^{5/2}$

d] $x^{3/2}(x + x^{5/2} - x^2)$ $x^{5/2} + x^4 - x^{7/2}$ e] $\frac{5-x}{x^2-25}$ $\frac{-1}{x+5}$

3] Expand using **PASCAL'S TRIANGLE**. $(x - 2y)^5$

$$x^5 - 10x^4y + 40x^3y^2 - 80x^2y^3 + 80xy^4 - 32y^5$$

4] **FUNCTIONS:** Let $f(x) = x^2$, $g(x) = 2x + 5$, $h(x) = x^2 - 1$. Find each.

a] $h[f(-2)] = 15$ b] $f[g(x-1)] = 4x^2 + 12x + 9$ c] $g[h(x^3)] = 2x^6 + 3$

Find $\frac{f(x+h)-f(x)}{h}$ for the given function, $f(x)$.

d] $f(x) = 9x + 3$ = 9

e] $f(x) = 5 - 2x$ = -2

5] **INTERCEPTS:** For the x-intercepts and y-intercepts for each.

a] $y = x^2 + x - 2$ $x = -2, x = 1, y = -2$

b] $y = x\sqrt{16 - x^2}$ $x = -4, x = 4, y = 0$


6] **POINTS OF INTERSECTION:** Find the point(s) of intersection of the graphs algebraically.


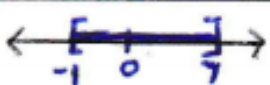
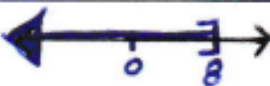
a] $x + y = 8$ and $4x - y = 7$ (3, 5)

b] $x^2 + y = 6$ and $x + y = 4$ (2, 2) and (-1, 5)

7] **INTERVAL NOTATION, SET-BUILDER NOTATION, INEQUALITIES, & GRAPHS**

Complete the table with the appropriate notation or graph.

Solution	Interval Notation	Graph
$-2 < x \leq 4$		
	$[-1, 7)$	
		

Solution	Interval Notation	Graph
$-2 < x \leq 4$	$(-2, 4]$	
$-1 \leq x < 7$	$[-1, 7)$	
$x \leq 8$	$(-\infty, 8]$	

8] DOMAIN AND RANGE

Find the domain and Range of each function. Write your answer in interval notation.

a) $f(x) = x^2 - 5$ D: $(-\infty, \infty)$; R: $[-5, \infty)$

b) $f(x) = -\sqrt{x+3}$ D: $[-3, \infty)$; R: $(-\infty, 0]$

c) $f(x) = 3 \sin(x)$ D: $(-\infty, \infty)$; R: $[-3, 3]$

d) $f(x) = \frac{2}{x-1}$ D: $(-\infty, 1) \cup (1, \infty)$; R: $(-\infty, 0) \cup (0, \infty)$

9] INVERSE OF A FUNCTION: Find the inverse for each function.

a) $f(x) = 2x + 1$ $f^{-1}(x) = \frac{x-1}{2}$ b) $f(x) = \frac{x^2}{3}$ $f^{-1}(x) = \pm\sqrt{3x}$

Prove $f(x)$ and $g(x)$ are inverses of each other using compositions.

c) $f(x) = \frac{x^3}{2}$ and $g(x) = \sqrt[3]{2x}$ $f[g(x)] = g[f(x)] = x$

d) $f(x) = 9 - x^2, x \geq 0$ and $g(x) = \sqrt{9-x}$ $f[g(x)] = g[f(x)] = x$

10] **EQUATIONS OF LINES**

a] Determine the equation of a line passing through (5, -3) with an undefined slope.

$$x = 5$$

b] Determine the equation of a line passing through (-4, 2) with a slope of 0.

$$y = 2$$

c] Find the equation of a line passing through (2, 8) and perpendicular to $y = \frac{5}{6}x - 1$.

$$y = \frac{5}{6}x + \frac{19}{3}; 5x - 6y = -38$$

d] Find the equation of a line passing through (0, 5) and parallel to a line with a slope of 2/3.

$$y - 5 = \frac{2}{3}(x - 0)$$

e] Find the equation of a line with an x-intercept of (2, 0) and a y-intercept of (0, 3).

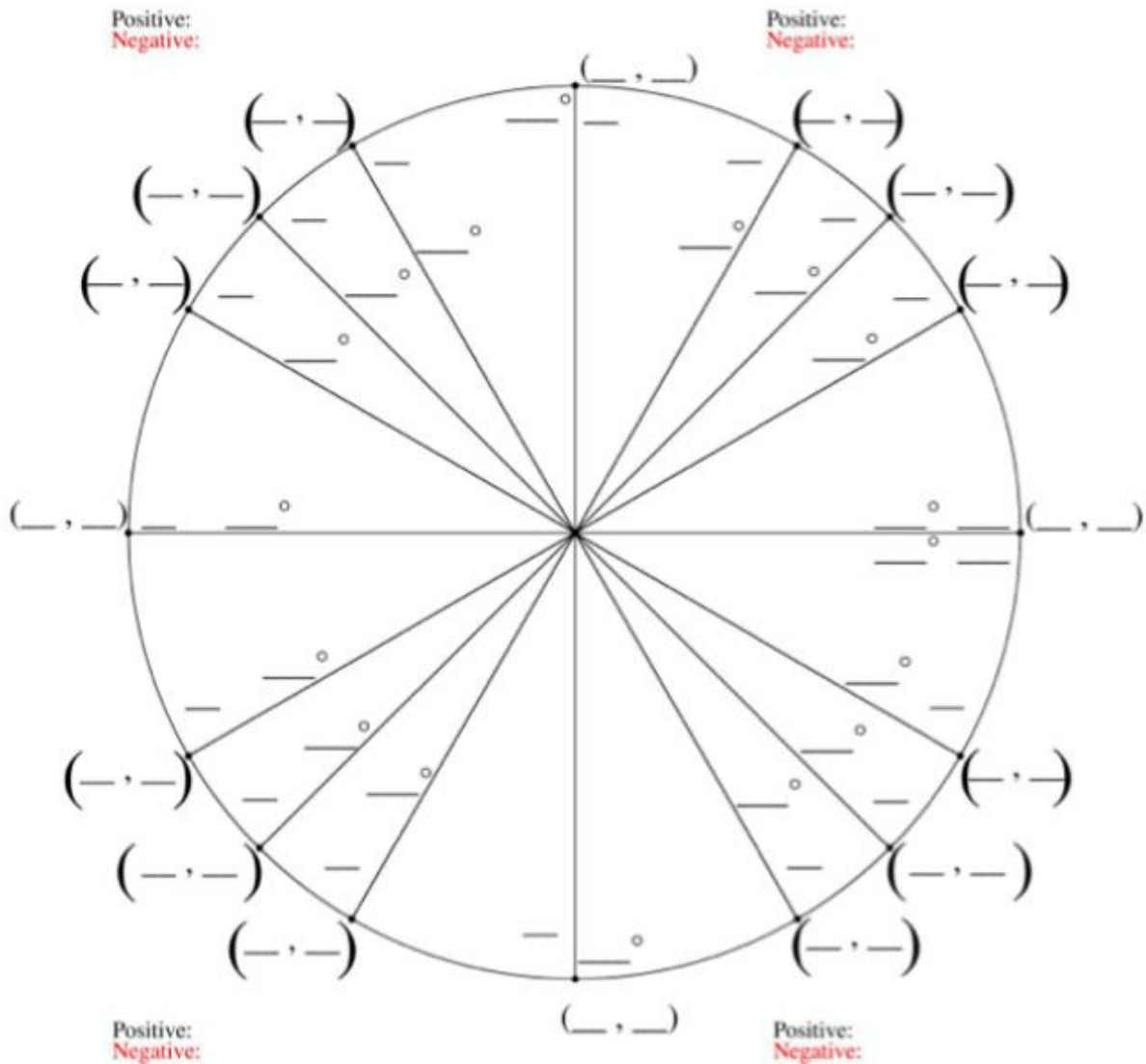
$$y = -\frac{3}{2}x + 3; 3x + 2y = 6$$

11] **RADIAN & DEGREE MEASURES**

Convert to degrees. a] $\frac{5\pi}{6}$ 150° b] $\frac{4\pi}{5}$ 144°

Convert to radians. c] 45° $\frac{\pi}{4}$ d] -17° $\frac{-17\pi}{180}$ e] 237° $\frac{79\pi}{60}$

12] Fill in the unit circle with the ordered pair, degree, and radian.



13] UNIT CIRCLE

You must have these memorized OR know how to calculate their values without a calculator.

- a] $\sin(\pi)$ b] $\cos\left(\frac{3\pi}{2}\right)$ c] $\sin\left(-\frac{\pi}{2}\right)$ d] $\sin\left(\frac{5\pi}{4}\right)$ e] $\cos\left(\frac{\pi}{4}\right)$
 f] $\cos(-\pi)$ g] $\cos\left(\frac{\pi}{3}\right)$ h] $\sin\left(\frac{5\pi}{6}\right)$ i] $\cos\left(\frac{2\pi}{3}\right)$ j] $\tan\left(\frac{\pi}{4}\right)$
 k] $\tan(\pi)$ l] $\tan\left(\frac{\pi}{3}\right)$ m] $\cos\left(\frac{4\pi}{3}\right)$ n] $\sin\left(\frac{11\pi}{6}\right)$ o] $\tan\left(\frac{7\pi}{4}\right)$ p] $\sin\left(-\frac{\pi}{6}\right)$

- a] 0 b] 0 c] -1 d] $-\frac{\sqrt{2}}{2}$ e] $\frac{\sqrt{2}}{2}$ f] -1
 g] $\frac{1}{2}$ h] $\frac{1}{2}$ i] $-\frac{1}{2}$ j] 1 k] 0 l] $\sqrt{3}$
 m] $-\frac{1}{2}$ n] $-\frac{1}{2}$ o] -1 p] $-\frac{1}{2}$

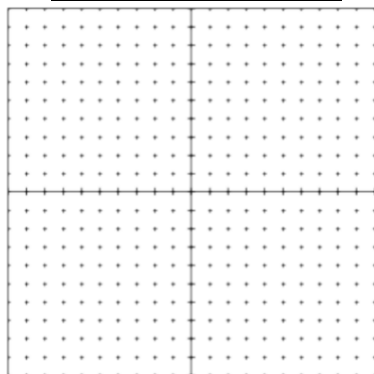
14] **TRIGONOMETRIC EQUATIONS:** Solve each of the equations for $0 \leq x < 2\pi$.

a] $\sin(x) = -1/2$ $\frac{7\pi}{6}, \frac{11\pi}{6}$ b] $2 \cos(x) = \sqrt{3}$ $\frac{\pi}{6}, \frac{11\pi}{6}$

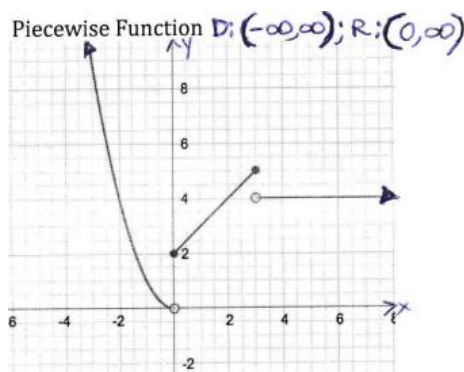
c] $4\sin^2 x = 3$ $\frac{\pi}{3}, \frac{2\pi}{3}, \frac{4\pi}{3}, \frac{5\pi}{3}$

d] $2\cos^2 x - 1 - \cos x = 0$ (Hint: Factor) $0, \frac{2\pi}{3}, \frac{4\pi}{3}, 2\pi$

15] **PIECEWISE FUNCTION:** Graph the function. Indicate the domain and range.



$$f(x) = \begin{cases} x^2, & x < 0 \\ x + 2, & 0 \leq x \leq 3 \\ 4, & x > 3 \end{cases}$$



16] **TRANSFORMATIONS**

a] Given $f(x) = x^2$ and $g(x) = (x - 3)^2 + 1$. Describe the transformations.
 $g(x)$ is $f(x)$ vertically translated 1 unit up and horizontally translated 3 units right

b] Write a new function, $g(x)$, for $f(x) = x^3$ translated six units left and reflected over the x-axis.

$$g(x) = -(x + 6)^3$$

c] If the ordered pair (2, 4) is on the graph of $f(x)$, find one ordered pair that will be on the following functions:

- i] $f(x) - 3$ (2, 1) ii] $f(x - 3)$ (5, 4) iii] $2f(x)$ (2, 8)
 iv] $f(x - 2) + 1$ (4, 5) v] $-f(x)$ (2, -4)

17] **EXPONENTIAL FUNCTIONS:** Solve for x .

a] $3^{3x+5} = 9^{2x+1}$ $x = 3$

b] $\left(\frac{1}{9}\right)^x = 27^{2x+4}$ $x = -3/2$

c] $\left(\frac{1}{6}\right)^x = 216$ $x = -3$

- 18] **LOGARITHMS:** Evaluate. a) $\log_7 7 = 1$ b) $\log_3 27 = 3$ c) $\log_2 \left(\frac{1}{32}\right) = -5$
d) $\log_{25} 5 = 1/2$ e) $\log_9 1 = 0$ f) $\log_4 8 = 3/2$
g) $\ln \sqrt{e} = 1/2$ h) $\ln \left(\frac{1}{e}\right) = -1$

19] **PROPERTIES OF LOGARITHMS:** Use the properties of logarithms to evaluate the following.

- a) $\log_2 2^5 = 5$ b) $\ln e^3 = 3$ c) $\log_2 8^3 = 9$
d) $\log_3 \sqrt[5]{9} = 2/5$ e) $2^{\log_2 10} = 10$ f) $e^{\ln 8} = 8$
g) $9 \ln e^2 = 18$ h) $\log_9 9^3 = 3$ i) $\log_{10} 25 + \log_{10} 4 = 2$
j) $\log_2 40 - \log_2 5 = 3$ k) $\log_2 (\sqrt{2})^5 = 5/2$

20] Solve for x.

- a) $\ln(e^3) = x$ b) $\ln(e^x) = 4$ c) $\ln(x) + \ln(x) = 0$
d) $e^{\ln 5} = x$ e) $\ln(1) - \ln(e) = x$ f) $\ln(6) + \ln(x) - \ln(2) = 3$
g) $\ln(x+5) = \ln(x-1) - \ln(x+1)$

- a] 3 b] 4 c] 1 d] 5 e] -1 f] $\frac{e^3}{3}$ g] -2; -3

21] **EVEN AND ODD FUNCTIONS**

State whether the graphs are even, odd, or neither. Show your work.

- a) $f(x) = 2x^4 - 5x^2$ even b) $g(x) = x^5 - 3x^3 + x$ odd
c) $h(x) = 2x^2 - 5x + 3$ neither d) $j(x) = 2\cos(x)$ even
e) $k(x) = \sin(x) + 4$ neither

22] **VERTICAL ASYMPTOTES:** Determine all vertical asymptotes.

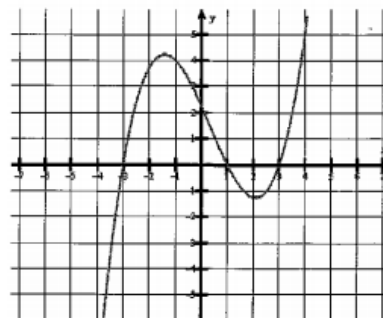
- a) $f(x) = \frac{1}{x^2}$ $x = 0$ b) $f(x) = \frac{x^2}{x^2-4}$ $x = -2, x = 2$
d) $f(x) = \frac{x-1}{x^2+x-2}$ $x = -2$

23] **HORIZONTAL ASYMPTOTES:** Determine all horizontal asymptotes.

- a) $f(x) = \frac{x^2-2x+1}{x^3+x-7}$ $y = 0$ b) $f(x) = \frac{5x^3-2x^2+8}{4x-3x^3+5}$ $y = -5/3$

24] The graph of the function f is given.

- A] State the domain of f . $(-\infty, \infty)$
B] State the range of f . $(-\infty, \infty)$
C] Estimate the values of x such that $f(x)=0$. $x = -3, 1, 3$
D] On which intervals is f increasing? $(-\infty, -1.5), (2, \infty)$
E] On which intervals is f decreasing? $(-1.5, 2)$



25] **PARENT FUNCTIONS**