AP CALCULUS AB SUMMER MATH 2019 - KEY

1] **COMPLEX FRACTIONS**: Simplify each of the following.

a] $\frac{\frac{25}{a}-a}{5+a}$ $\quad \frac{5-a}{a}$ b] $\frac{4-\frac{12}{2x-3}}{5+\frac{15}{2x-3}}$ $\quad \frac{4(x-3)}{5x}$ c] $\frac{\frac{x}{x+1}-\frac{1}{x}}{\frac{x}{x+1}+\frac{1}{x}}$ $\quad \frac{x^2-x-1}{x^2+x+1}$

2] **SIMPLIFYING EXPRESSIONS**: Write answers with positive exponents only.

a] $\frac{\frac{2}{x^2}}{\frac{10}{x^3}}$ $\quad \frac{x}{5}$ b] $\frac{12x^{-3}y^2}{18xy^{-1}}$ $\quad \frac{2y^3}{3x^4}$ c] $\left(4a^{\frac{5}{3}}\right)^{\frac{3}{2}}$ $\quad 8a^{5/2}$

d] $x^{\frac{3}{2}}\left(x+x^{\frac{5}{2}}-x^2\right)$ $\quad x^{5/2}+x^4-x^{7/2}$ e] $\frac{5-x}{x^2-25}$ $\quad \frac{-1}{x+5}$

3] Expand using **PASCAL'S TRIANGLE**. $(x-2y)^5$

$x^5-10x^4y+40x^3y^2-80x^2y^3+80xy^4-32y^5$

4] **FUNCTIONS**: Let $f(x)=x^2$, $g(x)=2x+5$, $h(x)=x^2-1$. Find each.

a] $h[f(-2)]$ = 15 b] $f[g(x-1)]$ $\quad 4x^2+12x+9$ c] $g[h(x^3)]$ $\quad 2x^6+3$

Find $\frac{f(x+h)-f(x)}{h}$ for the given function, f(x).
d] $f(x)=9x+3$ = 9
e] $f(x)=5-2x$ = -2

5] **INTERCEPTS**: For the x-intercepts and y-intercepts for each.

a] $y=x^2+x-2$ x = -2, x = 1, y= -2

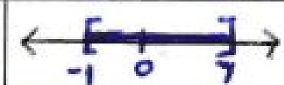
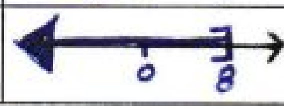
b] $y=x\sqrt{16-x^2}$ x = -4, x = 4, y = 0

c] $y^2=x^3-4x$ $x=\pm 2, x=0, y=0$

6] **POINTS OF INTERSECTION**: Find the point(s) of intersection of the graphs algebraically.
a] $x+y=8$ *and* $4x-y=7$ (3, 5)
b] $x^2+y=6$ *and* $x+y=4$ (2,2) and (-1, 5)

7] **INTERVAL NOTATION, SET-BUILDER NOTATION, INEQUALITIES, & GRAPHS**
Complete the table with the appropriate notation or graph.

Solution	Interval Notation	Graph
$-2 < x \le 4$		
	$[-1, 7)$	
		

Solution	Interval Notation	Graph
$-2 < x \le 4$	$(-2, 4]$	
$-1 \le x < 7$	$[-1, 7)$	
$x \le 8$	$(-\infty, 8]$	

8] **DOMAIN AND RANGE**
Find the domain and Range of each function. Write your answer in interval notation.

a] $f(x) = x^2 - 5$ D: $(-\infty, \infty)$; R: $[-5, \infty)$

b] $f(x) =- \sqrt{x+3}$ D: $[-3, \infty)$; R: $(-\infty, 0]$

c] $f(x) = 3\sin sin\ (x)$ D: $(-\infty, \infty)$; R: $[-3, 3]$

d] $f(x) = \frac{2}{x-1}$ D: $(-\infty, 1) \cup (1, \infty)$; R: $(-\infty, 0) \cup (0, \infty)$

9] **INVERSE OF A FUNCTION**: Find the inverse for each function.

a] $f(x) = 2x + 1$ $f^{-1}(x) = \frac{x-1}{2}$ b] $f(x) = \frac{x^2}{3}$ $f^{-1}(x) = \pm\sqrt{3x}$

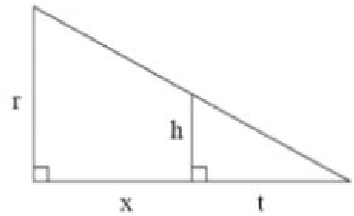
Prove f(x) and g(x) are inverses of each other using compositions.

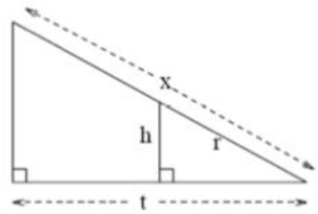
c] $f(x) = \frac{x^3}{2}$ and $g(x) = \sqrt[3]{2x}$ $f[g(x)] = g[f(x)] = x$

d] $f(x) = 9 - x^2,\ x \ge 0$ and $g(x) = \sqrt{9-x}$ $f[g(x)] = g[f(x)] = x$

10] **SIMILAR TRIANGLES**

Express x in terms of the other variables in the picture.

a] 

b] 

$x = \frac{rt - th}{h} = \frac{rt}{h} - t$

$x = \frac{tr}{\sqrt{r^2 - h^2}}$

11] **EQUATIONS OF LINES**

a] Determine the equation of a line passing through (5, -3) with an undefined slope.

x = 5

b] Determine the equation of a line passing through (-4, 2) with a slope of 0.

y = 2

c] Find the equation of a line passing through (2, 8) and perpendicular to $y = \frac{5}{6}x - 1$.

$y = \frac{5}{6}x + \frac{19}{3}; 5x - 6y = -38$

d] Find the equation of a line passing through (0, 5) and parallel to a line with a slope of 2/3.

$y - 5 = \frac{2}{3}(x - 0)$

e] Find the equation of a line with an x-intercept of (2, 0) and a y-intercept of (0, 3).

$y = -\frac{3}{2}x + 3; 3x + 2y = 6$

12] **USING THE GRAPHING CALCULATOR**

You should be able to a) graph an arbitrary function, b) find zeros of a function, and c) find the intersection between two functions. Draw a sketch and indicate the window used.
Find all roots to the nearest thousandth.

a] $f(x) = x^4 - 3x^3 + 2x^2 - 7x - 11$ $\approx -0.911; \approx 3.329$

b] $f(x) = 3\sin sin\ (2x)\ - 4x + 1$, $[-2\pi,\ 2\pi]$ (Hint: For trig. functions use radian mode.)

≈ 0.957

c] $f(x) = 0.7x^2 + 3.2x + 1.5$ $\approx -0.530; \approx -4.041$

d] $f(x) = x^4 - 8x^2 + 5$ $\approx \pm 2.705; \approx \pm 0.827$

f] Find the coordinates of any points of intersection. $f(x) = x^2 - 5x + 2$, $g(x) = 3 - 2x$

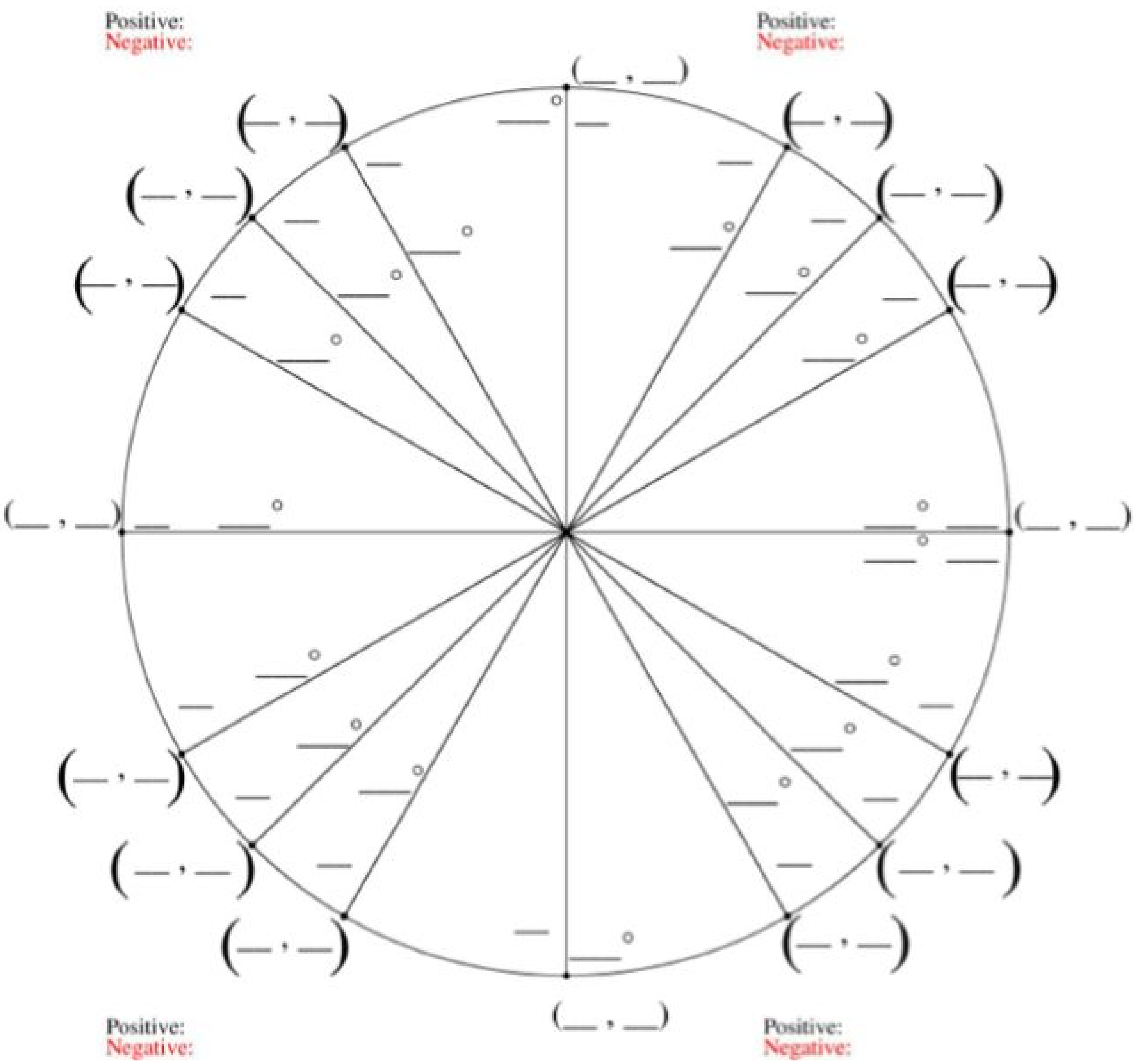
$(-0.303, 3.606); (3.303, -3.606)$

13] **RADIAN & DEGREE MEASURES**

Convert to degrees. a] $\frac{5\pi}{6}$ 150° b] $\frac{4\pi}{5}$ 144°

Convert to radians. c] 45° $\frac{\pi}{4}$ d] -17° $\frac{-17\pi}{180}$ e] 237° $\frac{79\pi}{60}$

14] Fill in the unit circle with the ordered pair, degree, and radian.



15] **UNIT CIRCLE**

You must have these memorized OR know how to calculate their values without a calculator.

a] $\sin sin\,(\pi)$ b] $\cos cos\,\left(\frac{3\pi}{2}\right)$ c] $\sin sin\,\left(-\frac{\pi}{2}\right)$ d] $\sin sin\,\left(\frac{5\pi}{4}\right)$ e] $\cos cos\,\left(\frac{\pi}{4}\right)$

f] $\cos cos\,(-\pi)$ g] $\cos cos\,\left(\frac{\pi}{3}\right)$ h] $\sin sin\,\left(\frac{5\pi}{6}\right)$ i] $\cos cos\,\left(\frac{2\pi}{3}\right)$ j] $\tan tan\,\left(\frac{\pi}{4}\right)$

k] $\tan tan\,(\pi)$ l] $\tan tan\,\left(\frac{\pi}{3}\right)$ m] $\cos cos\,\left(\frac{4\pi}{3}\right)$ n] $\sin sin\,\left(\frac{11\pi}{6}\right)$ o] $\tan tan\,\left(\frac{7\pi}{4}\right)$ p] $\sin sin\,\left(-\frac{\pi}{6}\right)$

a] 0 b] 0 c] -1 d] $-\frac{\sqrt{2}}{2}$ e] $\frac{\sqrt{2}}{2}$ f] -1

g] $\frac{1}{2}$ h] $\frac{1}{2}$ i] $-\frac{1}{2}$ j] 1 k] 0 l] $\sqrt{3}$

m] $-\frac{1}{2}$ n] $-\frac{1}{2}$ o] -1 p] $-\frac{1}{2}$

16] **TRIGONOMETRIC EQUATIONS**: Solve each of the equations for $0 \le x < 2\pi$.

a] $\sin sin\,(x) = -1/2$ $\frac{7\pi}{6};\ \frac{11\pi}{6}$ b] $2\cos cos\,(x) = \sqrt{3}$ $\frac{\pi}{6};\ \frac{11\pi}{6}$

c] $4sin^2x = 3$ $\frac{\pi}{3};\ \frac{2\pi}{3};\ \frac{4\pi}{3};\ \frac{5\pi}{3}$

d] $2cos^2x - 1 - cosx = 0$ (Hint: Factor) $0;\ \frac{2\pi}{3};\ \frac{4\pi}{3};\ 2\pi$

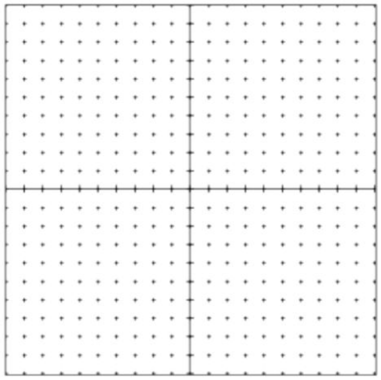
17] **PARENT FUNCTIONS**

Know the parent functions studied in Algebra 1, Algebra 2, and Pre-Calculus.

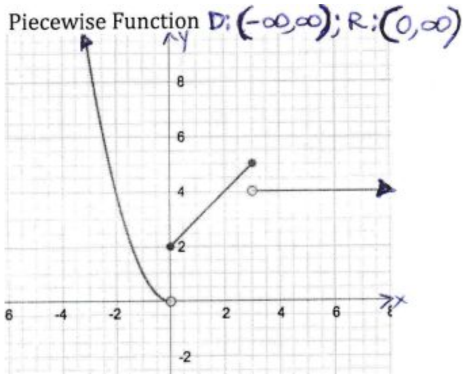
Know the key characteristics of each.

Identity	Linear	Quadratic	Cubic	Quartic	
Absolute Value	Square Root	Cubed Root	Exponential	Logarithmic	
Greatest Integer	Rational/Reciprocal	Piecewise	Quintic	Cotangent	
Sine	Cosine	Tangent	Secant	Cosecant	Constant

18] **PIECEWISE FUNCTION**: Graph the function. Indicate the domain and range.



$f(x) = \{x^2,\ x < 0\ x + 2,\ \ 0 \leq x \leq 3\ 4,\ x > 3$



19] **TRANSFORMATIONS**

a] Given $f(x) = x^2$ and $g(x) = (x-3)^2 + 1$. Describe the transformations.

g(x) is f(x) vertically translated 1 unit up and horizontally translated 3 units right

b] Write a new function, g(x), for $f(x) = x^3$ translated six units left and reflected over the x-axis.

$$g(x) = -(x+6)^3$$

c] If the ordered pair (2, 4) is on the graph of f(x), find one ordered pair that will be on the following functions:

i] f(x) – 3 (2, 1) ii] f(x – 3) (5, 4) iii] 2f(x) (2, 8)

iv] f(x – 2) + 1 (4, 5) v] -f(x) (2, -4)

20] **EXPONENTIAL FUNCTIONS**: Solve for x.

a] $3^{3x+5} = 9^{2x+1}$ x = 3

b] $\left(\frac{1}{9}\right)^x = 27^{2x+4}$ x = -3/2

c] $\left(\frac{1}{6}\right)^x = 216$ x = -3

21] **LOGARITHMS**: Evaluate. a] 7 = 1 b] 27 = 3 c] $\left(\frac{1}{32}\right)$ = -5

d] 5 = 1/2 e] 1 = 0 f] 8 = 3/2

g] $\ln ln \sqrt{e}$ = 1/2 h] $\ln ln \left(\frac{1}{e}\right)$ = -1

22] **PROPERTIES OF LOGARITHMS**: Use the properties of logarithms to evaluate the following.

a] 2^5 = 5 b] $\ln ln\, e^3$ = 3 c] 8^3 = 9

d] $\sqrt[5]{9}$ = 2/5 e] 2^{10} = 10 f] $e^{\ln ln\, 8}$ = 8

g] e^2 = 18 h] 9^3 = 3 i] $25 + 4$ = 2

j] $40 - 5$ = 3 k] $\left(\sqrt{2}\right)^5$ = 5/2

23] Solve for x.

a] $ln\,(e^3) = x$ b] $ln\,(e^x) = 4$ c] $\ln ln\ (x) + \ln ln\ (x) = 0$

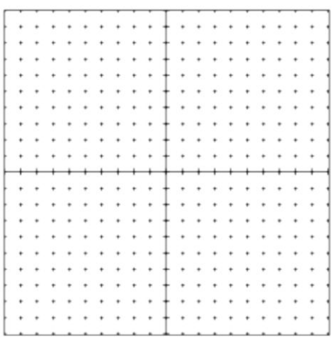
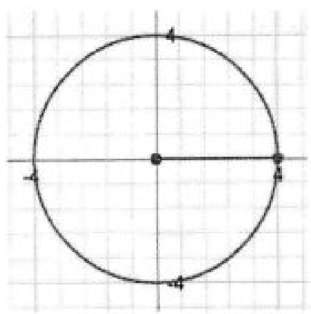
d] $e^{\ln ln\, 5} = x$ e] $\ln ln\ (1) - \ln ln\ (e) = x$ f] ln(6) + ln(x) – ln(2) = 3

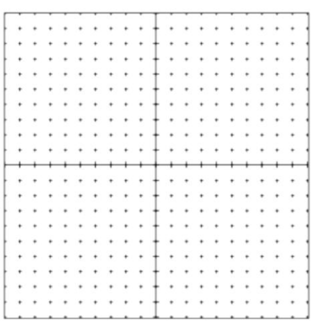
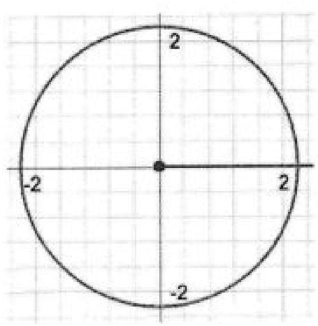
g] ln(x+5) = ln(x-1) – ln(x+1)

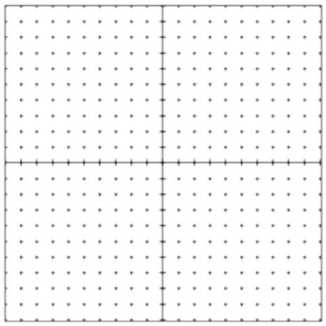
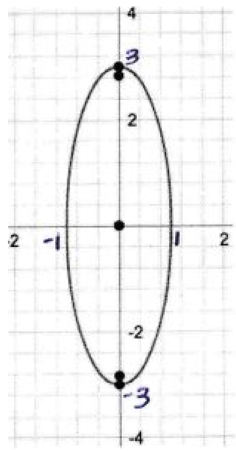
a] 3 b] 4 c] 1 d] 5 e] -1 f] $\frac{e^3}{3}$ g] -2; -3

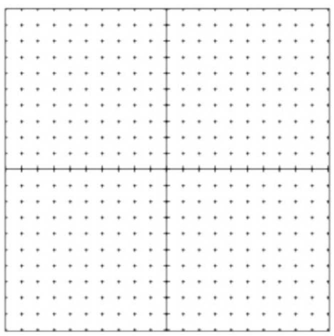
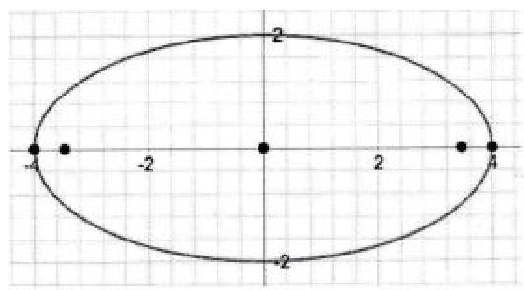
24] **CIRCLES AND ELLIPSES**: Graph.

a] $x^2 + y^2 = 16$ b] $x^2 + y^2 = 5$ c] $\frac{x^2}{1} + \frac{y^2}{9} = 1$ d] $\frac{x^2}{16} + \frac{y^2}{4} = 1$

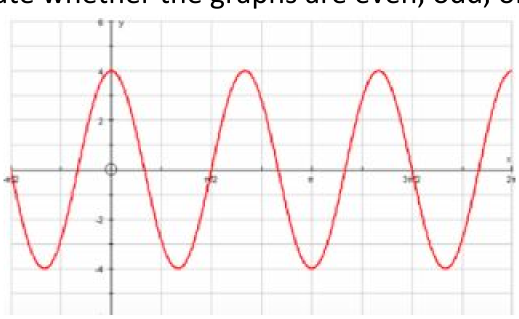
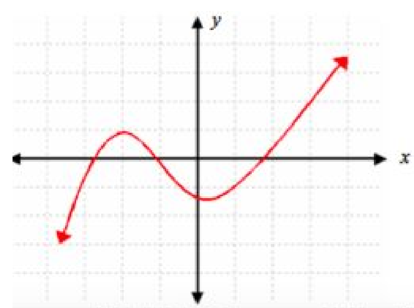
a]  

b]  

c] 


d] 


25] **<u>EVEN AND ODD FUNCTIONS</u>**
State whether the graphs are even, odd, or neither.

a]  even b] 

neither

State whether the graphs are even, odd, or neither. Show your work.

c] $f(x) = 2x^4 - 5x^2$ even
d] $g(x) = x^5 - 3x^3 + x$ odd
e] $h(x) = 2x^2 - 5x + 3$ neither
f] $j(x) = 2cos(x)$ even
g] $k(x) = \sin sin\ (x)\ + 4$ neither
h] $l(x) = \cos cos\ (x)\ - 3$ even

26] **<u>VERTICAL ASYMPTOTES</u>**: Determine all vertical asymptotes.

a] $f(x) = \frac{1}{x^2}$ x = 0
b] $f(x) = \frac{x^2}{x^2-4}$ x = -2, x = 2
c] $f(x) = \frac{2+x}{x^2(1-x)}$ x = 0, x = 1
d] $f(x) = \frac{x-1}{x^2+x-2}$ x = -2

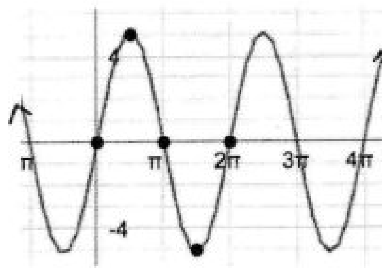
27] **HORIZONTAL ASYMPTOTES**: Determine all horizontal asymptotes.

a] $f(x) = \frac{x^2-2x+1}{x^3+x-7}$ y = 0 b] $f(x) = \frac{5x^3-2x^2+8}{4x-3x^3+5}$ y = -5/3

c] $f(x) = \frac{(2x-5)^2}{x^2-x}$ y = 4

28] **TRIGONOMETRIC FUNCTIONS**: Graph two complete periods of each function.

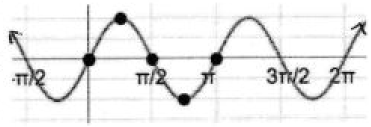
a] f(x) = 5sin(x)

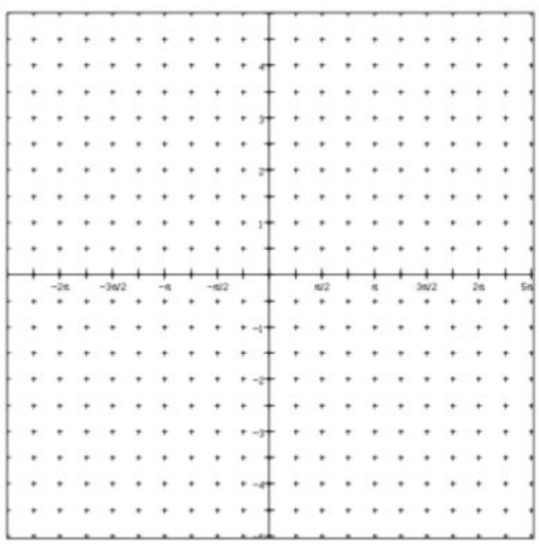


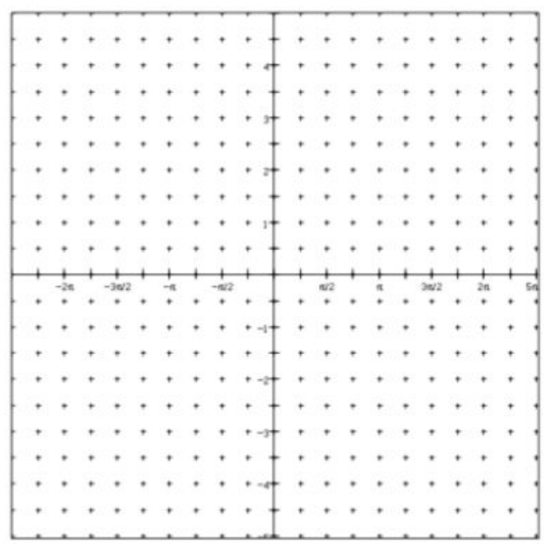
x	$f(x)$
0	0
$\frac{\pi}{2}$	5
π	0
$\frac{3\pi}{2}$	-5
2π	0

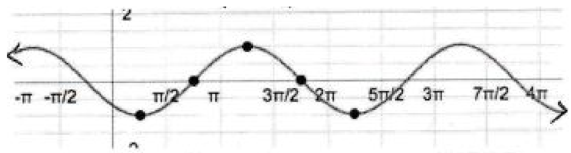
b] f(x) = sin(2x)

x	$f(x)$
0	0
$\frac{\pi}{4}$	1
$\frac{\pi}{2}$	0
$\frac{3\pi}{4}$	-1
π	0









c] $f(x) = -\cos\left(x - \frac{\pi}{4}\right)$

x	$f(x)$
$\frac{\pi}{4}$	-1
$\frac{3\pi}{4}$	0
$\frac{5\pi}{4}$	1
$\frac{7\pi}{4}$	0
$\frac{9\pi}{4}$	-1

Amplitude: 1

Period: 2π

Phase Shift: $\frac{\pi}{4}$ ($\frac{\pi}{4}$ to the right)

Vertical Shift: 0

d] f(x) = cos(x) – 3

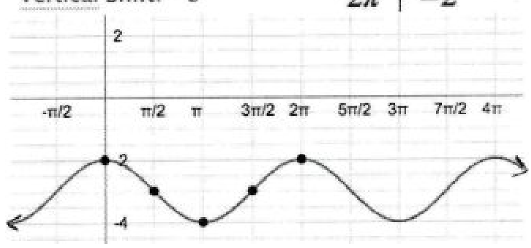
x	$f(x)$
0	-2
$\frac{\pi}{2}$	-3
π	-4
$\frac{3\pi}{2}$	-3
2π	-2

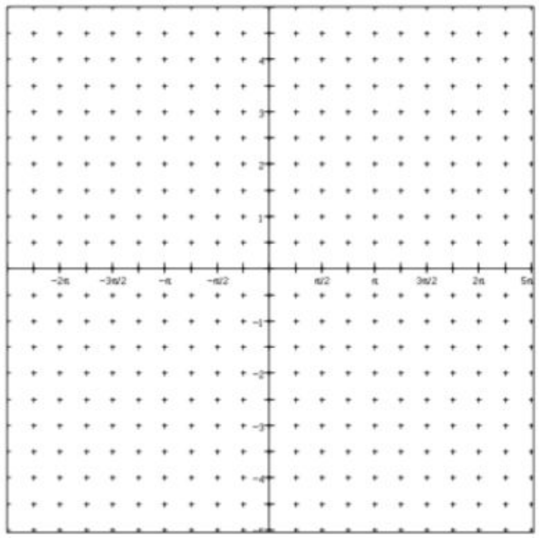
Amplitude: 1

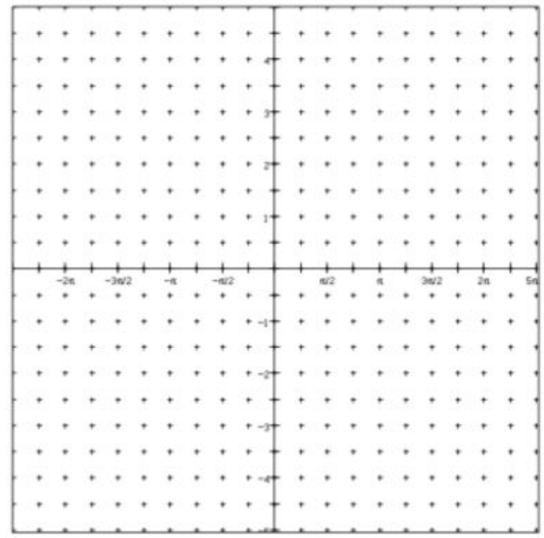
Period: 2π

Phase Shift: 0 (0 to the right)

Vertical Shift: -3




−2π
−3π/2
−π
−π/2
π/2
π
3π/2
2π
5π


−2π
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