NAME: ______________________________ DATE: ________ CLASS: ________

AP CALCULUS AB SUMMER MATH 2019 – REFERENCE SHEET

You may find some of the websites useful. For certain information, you may want to print and keep a copy in your notebook for future reference.

http://tutorial.math.lamar.edu/pdf/Trig_Cheat_Sheet.pdf
http://www.mathbits.com/MathBits/TeacherResources/PreCalculus/Formula%20Sheet2.pdf
http://www.khanacademy.org
http://www.math.ucdavis.edu/~marx/precalculus.html
http://justmathtutoring.com/
http://jamesrahn.com/
http://www.stewartcalculus.com/media/4_home.php
http://www.wtamu.edu/academic/anns/mps/math/mathlab/col_algebra/index.htm

COMMON MISTAKES:

Remember that $\sqrt{x^2+y^2} \neq x+y$. For example $\sqrt{2^2+3^2} \neq 2+3$ because $\sqrt{2^2+3^2} = \sqrt{13}$. Similarly, $\sqrt{x^2-y^2} \neq x-y$. However, if x and y are nonnegative, $\sqrt{x^2y^2} = xy$ and $\sqrt{\frac{x^2}{y^2}} = \frac{x}{y}$.

In a similar vein, $(x+y)^2 \neq x^2+y^2$ because $(x+y)^2 = (x+y)(x+y) = x^2+2xy+y^2$. However, $(xy)^2 = x^2y^2$ because $(xy)^2 = (xy)(xy) = xxyy = x^2y^2$. Also $(x-y)^2 \neq x^2-y^2$ but $\left(\frac{x}{y}\right)^2 = \frac{x^2}{y^2}$.

Care needs to be taken when deciding if canceling is possible. We have that $\frac{x+y}{x} \neq y$ but $\frac{xy}{x} = y$ because $\frac{xy}{x} = \left(\frac{x}{x}\right)y = (1)y = y$.

Recall that, for inequalities, if we multiply or divide both sides by a negative number, the inequality sign gets changed. We know that $-3 \leq 2$ but if we multiply by -2, we get $(-3)(-2) \geq 2(-2)$ because $6 \geq -4$.

When simplifying complex fractions, multiply by a fraction (equal to 1) which has the numerator and denominator composed of the common denominator of all the denominators in the complex fraction.

Example:

$$\frac{-7-\frac{6}{x+1}}{\frac{5}{x+1}} = \frac{-7-\frac{6}{x+1}}{\frac{5}{x+1}} \cdot \frac{x+1}{x+1} = \frac{-7(x+1)-6}{5} = \frac{-7x-13}{5}$$

$$\frac{\frac{-2}{x}+\frac{3x}{x-4}}{5-\frac{1}{x-4}} = \frac{\frac{-2}{x}+\frac{3x}{x-4}}{5-\frac{1}{x-4}} \cdot \frac{x(x-4)}{x(x-4)} = \frac{-2(x-4)+3x(x)}{5(x)(x-4)-1(x)} = \frac{-2x+8+3x^2}{5x^2-20x-x} = \frac{3x^2-2x+8}{5x^2-21x}$$

Simplifying Expressions

Make sure you are very comfortable manipulating exponents, positive/negative, fractional. Also, know the relationship between exponents and radicals; the appropriate radical will "undo" an exponent.

Examples:

$\sqrt[3]{(2)^3} = 2$

$\sqrt[3]{8} = 8^{1/3} = 2$

$\left(\sqrt[10]{25}\right)^5 = \left(25^{1/10}\right)^5 = 25^{5/10} = 25^{1/2} = \sqrt{25} = 5$

$x^{-4} = \frac{1}{x^4}$

$\frac{1}{x^{-3}} = x^3$

$(3x)^{-2} = \frac{1}{(3x)^2} = \frac{1}{9x^2}$

$x^6x^5 = x^{11}$

$\frac{x^3}{x^9} = x^{-6} = \frac{1}{x^6}$

$\left(x^2\right)^3 = x^6$

Functions

To evaluate a function for a given value, simply plug the value into the function for x.

Recall: $(f \circ g)(x) = f(g(x))$ *OR* $f[g(x)]$ read "*f* of *g* of *x*" means: plug the inside function (in this case g(x)) in for x in the outside function (in this case, f(x)).

Example: Given $f(x) = 2x^2 + 1$ *and* $g(x) = x - 4$ find *f(g(x))*.

$$\begin{aligned} f(g(x)) &= f(x-4) \\ &= 2(x-4)^2 + 1 \\ &= 2(x^2 - 8x + 16) + 1 \\ &= 2x^2 - 16x + 32 + 1 \\ f(g(x)) &= 2x^2 - 16x + 33 \end{aligned}$$

Intercepts and Points of Intersection

To find the x-intercepts, let y = 0 in your equation and solve.
To find the y-intercepts, let x = 0 in your equation and solve.

Example: $y = x^2 - 2x - 3$

<u>$x-\text{int.}\ (Let\ y = 0)$</u>

$0 = x^2 - 2x - 3$

$0 = (x-3)(x+1)$

$x = -1\ or\ x = 3$

$x-\text{intercepts}\ (-1,0)\ and\ (3,0)$

<u>$y-\text{int.}\ (Let\ x = 0)$</u>

$y = 0^2 - 2(0) - 3$

$y = -3$

$y-\text{intercept}\ (0,-3)$

Use substitution or elimination method to solve the system of equations.
Example:

$x^2 + y^2 - 16x + 39 = 0$

$x^2 - y^2 - 9 = 0$

<u>Elimination Method</u>

$2x^2 - 16x + 30 = 0$

$x^2 - 8x + 15 = 0$

$(x-3)(x-5) = 0$

$x = 3\ and\ x = 5$

Plug $x = 3\ and\ x = 5$ into one original

$3^2 - y^2 - 9 = 0 \qquad 5^2 - y^2 - 9 = 0$

$-y^2 = 0 \qquad 16 = y^2$

$y = 0 \qquad y = \pm 4$

Points of Intersection $(5,4)$, $(5,-4)$ *and* $(3,0)$

<u>Substitution Method</u>

Solve one equation for one variable.

$y^2 = -x^2 + 16x - 39$ (1st equation solved for y)

$x^2 - (-x^2 + 16x - 39) - 9 = 0$ Plug what y^2 is equal to into second equation.

$2x^2 - 16x + 30 = 0$ (*The rest is the same as previous example*)

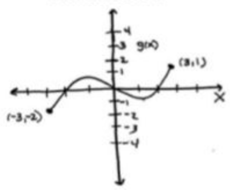
$x^2 - 8x + 15 = 0$

$(x-3)(x-5) = 0$

$x = 3\ or\ x - 5$

Domain – All x values for which a function is defined (input values)
Range – Possible y or Output values

EXAMPLE 1



a) Find Domain & Range of g(x).
The domain is the set of inputs of the function
Input values run along the horizontal axis.
The furthest left input value associated with a pt. on the graph is -3. The furthest right input values associated with a pt. on the graph is 3.
So Domain is [-3,3], that is all reals from -3 to 3.

The range represents the set of output values for the function. Output values run along the vertical axis. The lowest output value of the function is -2. The highest is 1. So the range is [-2,1], all reals from -2 to 1.

EXAMPLE 2

Find the domain and range of $f(x) = \sqrt{4 - x^2}$
Write answers in interval notation.

DOMAIN

For $f(x)$ to be defined $4 - x^2 \geq 0$.
This is true when $-2 \leq x \leq 2$
Domain: $[-2,2]$

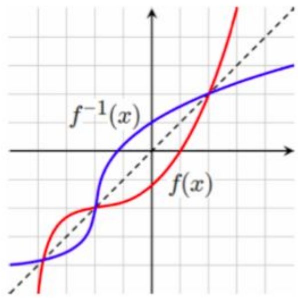
RANGE

The solution to a square root must always be positive thus $f(x)$ must be greater than or equal to 0.
Range: $[0,\infty)$

To find the inverse of a function, simply switch the x and the y and solve for the new "y" value.
Recall $f^{-1}(x)$ is defined as the inverse of $f(x)$

Example 1:

$f(x)=\sqrt[3]{x+1}$	Rewrite f(x) as y
$y=\sqrt[3]{x+1}$	Switch x and y
$x=\sqrt[3]{y+1}$	Solve for your new y
$(x)^3=\left(\sqrt[3]{y+1}\right)^3$	Cube both sides
$x^3=y+1$	Simplify
$y=x^3-1$	Solve for y
$f^{-1}(x)=x^3-1$	Rewrite in inverse notation



Also, recall that to PROVE one function is an inverse of another function, you need to show that:
$f(g(x))=g(f(x))=x$

Example:

If: $f(x)=\frac{x-9}{4}$ *and* $g(x)=4x+9$ **show *f(x) and g(x)* are inverses of each other.**

$$f(g(x))=4\left(\frac{x-9}{4}\right)+9$$
$$=x-9+9$$
$$=x$$

$$g(f(x))=\frac{(4x+9)-9}{4}$$
$$=\frac{4x+9-9}{4}$$
$$=\frac{4x}{4}$$
$$=x$$

$f(g(x))=g(f(x))=x$ therefore they are inverses of each other.

Slope intercept form: $y=mx+b$

Vertical line: x = c (slope is undefined)

Point-slope form: $y-y_1=m(x-x_1)$
* LEARN! We will use this formula frequently!

Horizontal line: y = c (slope is 0)

Example: Write a linear equation that has a slope of ½ and passes through the point (2, -6)

Slope intercept form

$y=\frac{1}{2}x+b$	Plug in ½ for m
$-6=\frac{1}{2}(2)+b$	Plug in the given ordered
$b=-7$	Solve for b
$y=\frac{1}{2}x-7$	

Point-slope form

$y+6=\frac{1}{2}(x-2)$	Plug in all variables
$y=\frac{1}{2}x-7$	Solve for y

Radian and Degree Measure

Use $\frac{180°}{\pi \, radians}$ to get rid of radians and convert to degrees.

Use $\frac{\pi \, radians}{180°}$ to get rid of degrees and convert to radians.

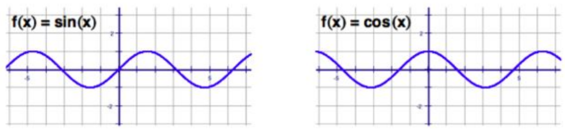
Circle with Radius 1

otherwise known as unit circle

$$(x,y) = (\cos\theta, \sin\theta)$$

You can determine the sine or cosine of an angle by using the unit circle. The x coordinate of the circle is cosine, and the y-coordinate is the sine.

Graphing Trig Functions



$y = \sin x$ and $y = \cos x$ have a period of 2π and an amplitude of 1. Use the parent graphs above to help you sketch a graph of the functions below. For $f(x) = A\sin(Bx + C) + K$, A = amplitude, $\frac{2\pi}{B}$ = period, $\frac{C}{B}$ = phase shift (positive C/B shift left, negative C/B shift right) and K = vertical shift.

Trigonometric Equations:

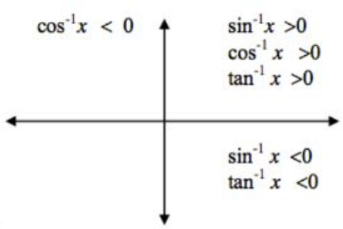
Solve each of the equations for $0 \le x < 2\pi$. Isolate the variable, sketch a reference triangle, find all the solutions within the given domain, $0 \le x < 2\pi$. Remember to double the domain when solving for a double angle. Use trig identities, if needed, to rewrite the trig functions. (See formula sheet at the end of the packet.)

Inverse Trigonometric Functions:

Recall: Inverse Trig Functions can be written in one of ways:

$$\arcsin(x) \qquad \sin^{-1}(x)$$

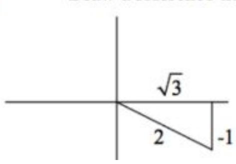
Inverse trig functions are defined only in the quadrants as indicated below due to their restricted domains.



Example:

Express the value of "y" in radians.

$y = \arctan\frac{-1}{\sqrt{3}}$ Draw a reference triangle.



This means the reference angle is 30° or $\frac{\pi}{6}$. So, $y = -\frac{\pi}{6}$ so that it falls in the interval from

$$\frac{-\pi}{2} < y < \frac{\pi}{2} \qquad \text{Answer: } y = -\frac{\pi}{6}$$

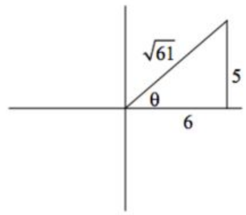
Example: Find the value without a calculator.

$\cos\left(\arctan\frac{5}{6}\right)$

Draw the reference triangle in the correct quadrant first.

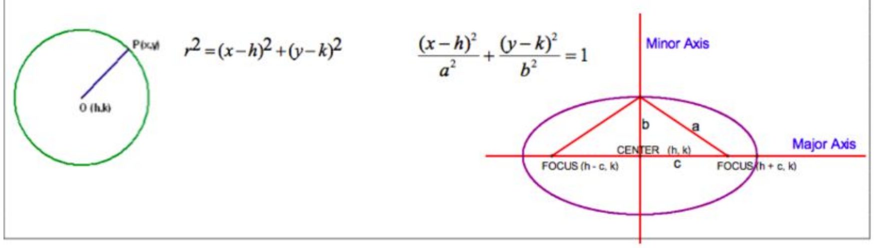
Find the missing side using Pythagorean Thm.

Find the ratio of the cosine of the reference triangle.



$$\cos\theta = \frac{6}{\sqrt{61}}$$

Circles and Ellipses



$r^2 = (x-h)^2 + (y-k)^2$

$\frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{b^2} = 1$

For a circle centered at the origin, the equation is $x^2 + y^2 = r^2$, where **r** is the radius of the circle.

For an ellipse centered at the origin, the equation is $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$, where **a** is the distance from the center to the ellipse along the x-axis and **b** is the distance from the center to the ellipse along the y-axis. If the larger number is under the y^2 term, the ellipse is elongated along the *y*-axis. For our purposes in Calculus, you will not need to locate the foci.

TRANSFORMATION OF FUNCTIONS

$h(x) = f(x) + c$	Vertical shift c units up	$h(x) = f(x-c)$	Horizontal shift c units right
$h(x) = f(x) - c$	Vertical shift c units down	$h(x) = f(x+c)$	Horizontal shift c units left
$h(x) = -f(x)$	Reflection over the x-axis		

EXPONENTIAL EQUATIONS

Example: Solve for x

$4^{x+1} = \left(\frac{1}{2}\right)^{3x-2}$

$\left(2^2\right)^{x+1} = \left(2^{-1}\right)^{3x-2}$	Get a common base
$2^{2x+2} = 2^{-3x+2}$	Simplify
$2x + 2 = -3x + 2$	Set exponents equal
$x = 0$	Solve for x

The statement $y = b^x$ can be written as $x = \log_b y$. They mean the same thing.

REMEMBER: A LOGARITHM IS AN EXPONENT

Recall $\ln x = \log_e x$

The value of e is 2.718281828… or $\lim_{x\to\infty}\left(1+\frac{1}{x}\right)^x$

Example: Evaluate the following logarithms

$\log_2 8 = ?$

In exponential for this is $2^? = 8$
Therefore $? = 3$
Thus $\log_2 8 = 3$

$\log_b xy = \log_b x + \log_b y$ $\quad$ $\log_b \frac{x}{y} = \log_b x - \log_b y$ $\quad$ $\log_b x^y = y\log_b x$ $\quad$ $b^{\log_b x} = x$

Examples:

Expand $\log_4 16x$	Condense $\ln y - 2\ln R$	Expand $\log_2 7x^5$
$\log_4 16 + \log_4 x$	$\ln y - \ln R^2$	$\log_2 7 + \log_2 x^5$
$2 + \log_4 x$	$\ln \frac{y}{R^2}$	$\log_2 7 + 5\log_2 x$

EVEN AND ODD FUNCTIONS

Recall:
Even functions *are functions that are symmetric over the y-axis.*
To determine algebraically we find out if $f(x) = f(-x)$
*(*Think about it what happens to the coordinate* $(x, f(x))$ *when reflected across the y-axis*)*

Odd functions *are functions that are symmetric about the origin.*
To determind algebraically we find out if $f(-x) = -f(x)$
*(*Think about it what happens to the coordinate* $(x, f(x))$ *when reflected over the origin*)*

Rational functions are ratios of polynomial functions. $h(x) = \frac{f(x)}{g(x)}$

$h(x)$ has a **zero** when $h(x)=0$ (which occurs when $f(x)=0$ and the factor does not cancel)

Ex. $h(x)=\frac{x^2+x-2}{x^2-1}$ $\quad h(x)=\frac{(x+2)(x-1)}{(x+1)(x-1)}$ $\quad h(x)=\frac{(x+2)\cancel{(x-1)}}{(x+1)\cancel{(x-1)}}$

Therefore, $h(x)=0$ when $x=-2$.

$h(x)$ has a **vertical asymptote** when $g(x)=0$ and the factor that causes $g(x)=0$ does not cancel

Ex. $h(x)=\frac{x^2+x-2}{x^2-1}$ $\quad h(x)=\frac{(x+2)(x-1)}{(x+1)(x-1)}$ $\quad h(x)=\frac{(x+2)\cancel{(x-1)}}{(x+1)\cancel{(x-1)}}$

Therefore, $h(x)$ has a vertical asymptote when $x=-1$.

$h(x)$ has a **hole** (is undefined but the limit exists) when $g(x)=0$ and the factor that causes $g(x)=0$ cancels from both $f(x)$ and $g(x)$.

Ex. $h(x)=\frac{x^2+x-2}{x^2-1}$ $\quad h(x)=\frac{(x+2)(x-1)}{(x+1)(x-1)}$ $\quad h(x)=\frac{(x+2)\cancel{(x-1)}}{(x+1)\cancel{(x-1)}}$

Therefore, $h(x)$ has a hole when $x=1$.

Note that $h(x)\neq\frac{(x+2)}{(x+1)}$ because these two functions do not have the same domain.

$h(x)$ has a **horizontal asymptote** at $y=a$ when $\lim_{x\to\infty}h(x)=a$ or $\lim_{x\to-\infty}h(x)=a$. To determine $\lim_{x\to\infty}h(x)$ consider first the largest exponent of $f(x)$ and $g(x)$. If $f(x)$ has the larger exponent, then $\lim_{x\to\infty}h(x)=\infty$ (DNE). If $g(x)$ has the larger exponent, then $\lim_{x\to\infty}h(x)=0$. If the exponents are the same, consider the leading coefficient.

Ex. $h(x)=\frac{x^2+x-2}{x^2-1}$ Leading coefficients $=\frac{1}{1}$

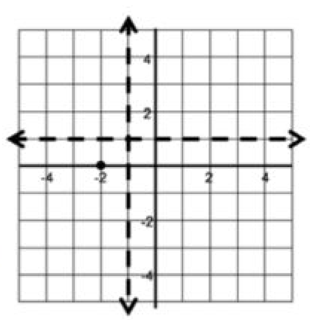
Therefore, $\lim_{x\to\infty}h(x)=1$ and $h(x)$ has a horizontal asymptote at $y=1$.

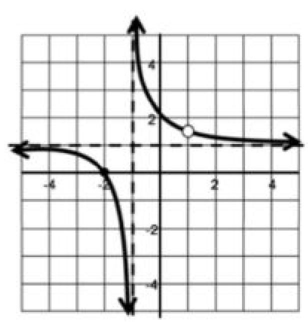
Once the basic characteristics of rational expressions are determined, the functions can be sketched without a calculator:

Ex. $h(x)=\frac{x^2+x-2}{x^2-1}$

Zero at $x=-2$.
Vertical Asymptote: $x=-1$
Hole when $x=1$
Horizontal Asymptote: $y=1$

Graph points as needed until you see the shape.

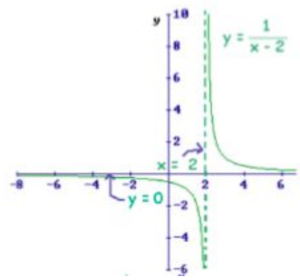




VERTICAL ASYMPTOTES

Determine the vertical asymptotes for the function. Set the denominator equal to zero to find the x-value for which the function is undefined. That will be the vertical asymptote given the numerator does not equal 0 also (Remember this is called removable discontinuity).
Write a vertical asymptotes as a line in the form x =

Example: Find the vertical asymptote of $y = \frac{1}{x-2}$
Since when $x = 2$ the function is in the form 1/0 then the vertical line $x = 2$ is a vertical asymptote of the function.



HORIZONTAL ASYMPTOTES

Determine the horizontal asymptotes using the three cases below.

Case I. Degree of the numerator is less than the degree of the denominator. The asymptote is y = 0.

Example: $y = \frac{1}{x-1}$ (As x becomes very large or very negative the value of this function will approach 0). Thus there is a horizontal asymptote at $y = 0$.

Case II. Degree of the numerator is the same as the degree of the denominator. The asymptote is the ratio of the lead coefficients.

Exmaple: $y = \frac{2x^2 + x - 1}{3x^2 + 4}$ (As x becomes very large or very negative the value of this function will approach 2/3). Thus there is a horizontal asymptote at $y = \frac{2}{3}$.

Case III. Degree of the numerator is greater than the degree of the denominator. There is no horizontal asymptote. The function increases without bound. (If the degree of the numerator is exactly 1 more than the degree of the denominator, then there exists a slant asymptote, which is determined by long division.)

Example: $y = \frac{2x^2 + x - 1}{3x - 3}$ (As x becomes very large the value of the function will continue to increase and as x becomes very negative the value of the function will also become more negative).

Reciprocal Identities: $\csc x = \frac{1}{\sin x}$ $\quad \sec x = \frac{1}{\cos x}$ $\quad \cot x = \frac{1}{\tan x}$

Quotient Identities: $\tan x = \frac{\sin x}{\cos x}$ $\quad \cot x = \frac{\cos x}{\sin x}$

Pythagorean Identities: $\sin^2 x + \cos^2 x = 1$ $\quad \tan^2 x + 1 = \sec^2 x$ $\quad 1 + \cot^2 x = \csc^2 x$

Double Angle Identities:

$$\sin 2x = 2\sin x \cos x$$

$$\tan 2x = \frac{2\tan x}{1 - \tan^2 x}$$

$$\cos 2x = \cos^2 x - \sin^2 x = 1 - 2\sin^2 x = 2\cos^2 x - 1$$